SUPPLEMENTAL INFORMATION

The firm seeks to maximize its profits given the following objective function, where d is the vector of product demands and f is the vector of product link flows, subject to the conservation of flow equations relating the path flows to the demands; the link flows to the path flows, and the non-negativity constraints on the path flows, along with the link labor and link flow expressions and the appropriate constraints under consideration on labor. The conservation of flow equations and the equation relating labor hours to product flow on a link are as follows.

$$\text{Maximize} \sum_{w \in W} \rho_w(d) d_w - \sum_{a \in L} \hat{c}_a(f) - \sum_{a \in L} \pi_a l_a, \tag{1}$$

The demand at a demand market is equal to the sum of the product flows of the firm to the demand market:

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W, \tag{2}$$

where x_p is the path flow on path p, d_w is the demand at w, and P_w is the set of paths from node 1 in Figure 1 to w.

The product flow on a link, f_a, is equal to the sum of flows on paths that contain that link:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and is 0, otherwise.

The path flows must all be nonnegative:

$$x_p \geq 0, \quad \forall p \in P, \tag{4}$$

where P is the set of all paths from node 1 in Figure 1 to the demand markets.

The following equation relating labor hours on a link a, l_a, with the product volume on a link must hold for each link:

$$f_a = \alpha_a l_a, \quad \forall a \in L. \tag{5}$$

There are different types of constraints that can be considered to capture labor disruptions. In this study, we consider, in the first and the third supply chain network examples, bounds on labor availability of the following form:

$$l_a \leq \bar{l}_a, \quad \forall a \in L, \tag{6}$$

whereas in the second, fourth, and fifth examples, we consider a looser bound of:

$$\sum_{a \in L} l_a \leq \bar{l}. \tag{7}$$

Details on the effective and efficient solution of such problems can be found in Nagurney (2021a).