Supplemental materials for "Betweenness centrality is not a resilience metric"

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Distributional effects

While the betweenness centrality maxima of the three networks presented are very similar, there do appear to be some differences between them in terms of the statistical and geographic distributions of centrality values. The distributions of betweenness centrality exhibit the same theoretical concerns about not considering non-shortest paths; herein, I examine whether there are empirical reasons to believe the distribution of centrality may be useful as a resilience metric. I determine that the distributional differences observed in the main paper are largely artifacts of the exact networks used, rather than generalizable findings about the relationship between centrality distributions and resilience.

Statistical distributions of betweenness centrality

Figure 1 shows the cumulative distribution functions of betweenness centrality for the three networks in the main paper. The Island and Short and Long Bridge networks have effectively identical cumulative distribution functions, despite clear differences in resilience. The Diagon Alley network shows more mid-level centrality values than the others.

However, this is very dependent on the exact location of the diagonal link. In Figure 2, I add gray lines showing the cumulative distribution of betweenness centrality for all possible locations of the diagonal shortcut within the Diagon Alley network, which shows that changing the location can make the distribution look much more similar to the other two networks.

In Figure 3, I compute the Kolmogorov-Smirnov p-values comparing the distribution of betweenness centrality for all possible locations of the diagonal shortcut to the original Diagon Alley network (light blue) and the Island network (dark blue). Unsurprisingly, the betweenness centrality distribution in the modified networks is in most cases not statistically distinguishable from the original Diagon Alley network. However, many of the modified networks also do not show a statistically-significant difference relative to the Island network.

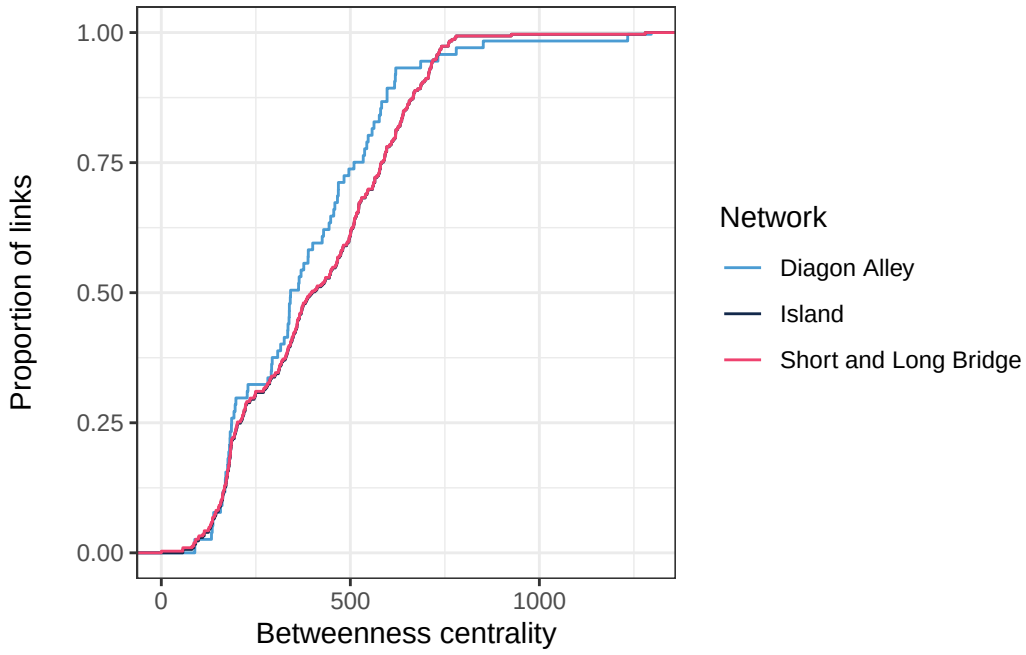

Figure 1: Cumulative distribution functions of betweenness centrality for the three networks presented in the article

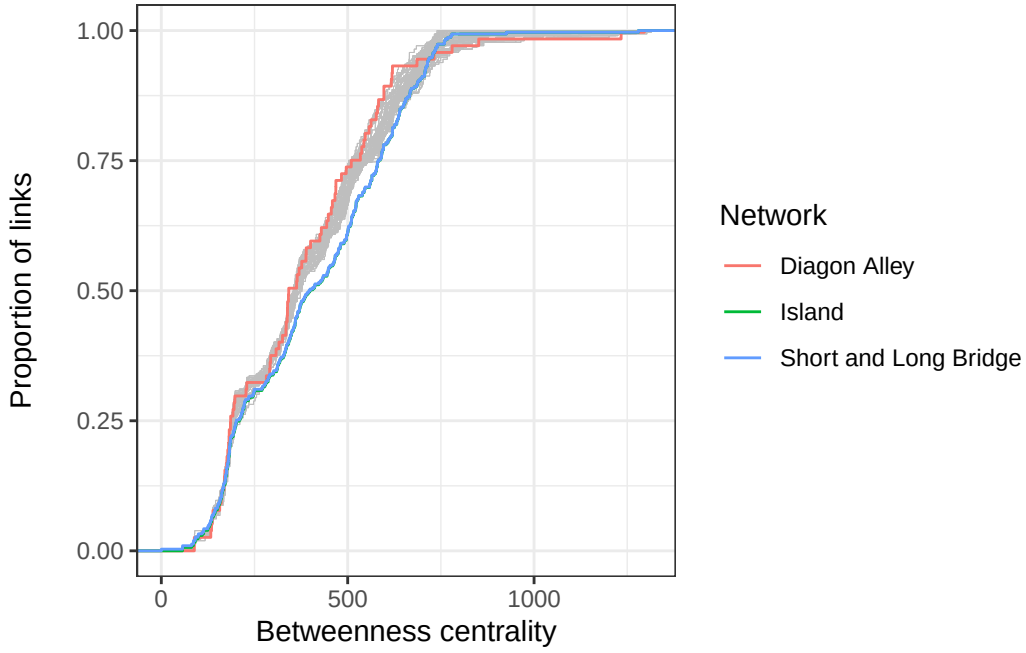

Figure 2: Cumulative distribution functions of betweenness centrality for the three networks presented in the article, with variations on Diagon Alley network

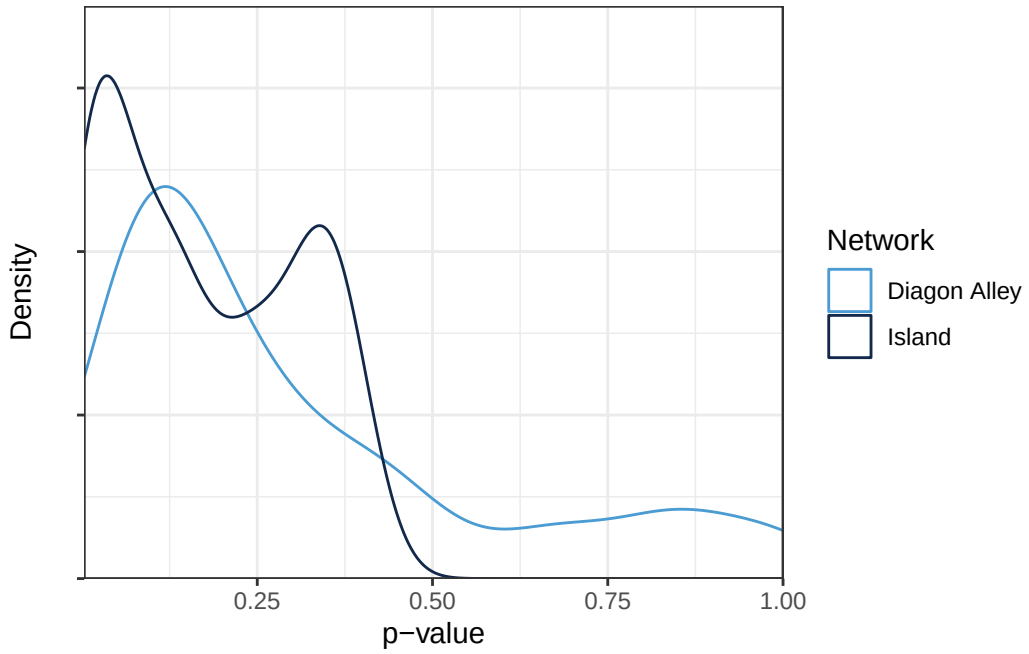

Figure 3: Cumulative distribution functions of betweenness centrality for the three networks presented in the article, with variations on Diagon Alley network

Geographic distributions of betweenness centrality

The geographic distribution of betweenness centrality also would initially seem to suggest resilience, with many similar betweenness centrality values in the center of the more resilient gridded parts of the networks. This is largely an artifact of the toy nature of the networks. These networks are perfect grids, where all links (other than the bridges and diagonal shortcut) are exactly one unit long, so most pairs of vertices have a multitude of equivalently-short paths between them. Figure 4 recalculates betweenness centrality after applying a small jitter to the locations of every vertex (up to 0.01 units in both X and Y directions) to better reflect real-world networks where lengths are not identical.[1] The jitter was applied consistently across networks—i.e. nodes in corresponding locations of different networks have identical jitter applied. The geographic concentration of mid-level links in the gridded areas network is essentially gone, and geographic concentrations look very similar across all networks. Thus, the geographic distribution also does not imply particular resilience outcomes.

[1]The two nodes connected to the long bridge are not jittered in any network to avoid having to recalculate the length of the curve, which was done manually.

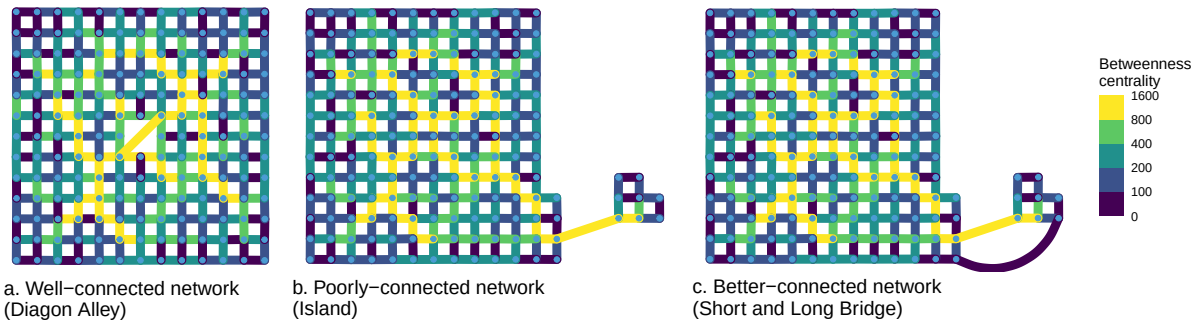

Figure 4: Betweenness centrality values for the three networks with jittering applied