Findings

Fagundes, Renan L., Yunfei Li, Fabiano L. Ribeiro, and Diego Rybski. 2025. "A Review of Percolation-like Transitions in Cities and Landscapes." *Findings*, December. https://doi.org/10.32866/001c.150358.

URBAN FINDINGS

A Review of Percolation-like Transitions in Cities and Landscapes

Renan L. Fagundes[1], Yunfei Li[1], Fabiano L. Ribeiro[2], Diego Rybski[1,3]

[1] Research Group Urban Complexity (UCP), Leibniz Institute of Ecological Urban and Regional Development (IOER), [2] Departamento de Fisica (DFI), Universidade Federal de Lavras (UFLA), [3] Complexity Science Hub Vienna

Keywords: percolation, phase transition, urban systems, landscape criticality, Review

https://doi.org/10.32866/001c.150358

Findings

We review papers analyzing percolation-like transitions in cities, urban systems, and landscapes. We organize their presentation according to (i) landscape ecology, (ii) city morphology and properties, (iii) modeling city growth, (iv) traffic dynamics, and (v) forests and vegetation. A closer look at the second group reveals that, overall, the works are limited to a small number of different control and order parameters. We hierarchically cluster the respective papers. Related to connectivity and fragmentation, percolation-like transitions arise in various complex, natural and artificial structures in cities and landscapes. More research is needed to better understand the differences between predictions from percolation theory and findings from real-world data.

In *percolation theory*, one studies the transition a structure can undergo between a connected and a fragmented phase, and its properties at and around this critical point. For an overview we refer to (Bunde and Havlin 1991; Stauffer and Aharony 1994; Isichenko 1992; Efros 1986, e.g.). For instance, randomly occupying the cells of a square grid, at the critical probability of, in this case, ≈0.59, abruptly a giant cluster emerges that extends from one side of the system to the other (Brockmann 2018). The *control parameter* is the variable that is being changed (e.g. the occupancy probability) and the transition is reflected in the *order parameter*, which is usually associated with macroscopic properties of the system (e.g. the probability that an occupied cell belongs to the giant cluster), and expected to diverge or become trivial (e.g., 1) at the transition. However, the strict theory applies only to the limit of infinite system size. In this example of occupying a grid, the cells are discrete. Another form is continuum percolation, where objects (typically circles) are placed in continuous space, eventually forming a percolating structure (Meester and Roy 2008; Balberg 2009; Mertens and Moore 2012, e.g.). While percolation theory has been developed using derivations and simulations of random processes, percolation-like transitions have also been observed in many empirical studies. Here we selectively review works carried out in the context of cities and urban systems and, more generally, in the context of landscape.

In landscape ecology, *neutral models* represent a sort of null hypothesis used to explain biodiversity patterns assuming all species are functionally equivalent (Gardner et al. 1987). The properties of these neutral models, including the number of clusters, size distribution, fractal dimension, separation between clusters, and correlation length (e.g. defined as mean distance between two sites on the same finite cluster, (Bunde and Havlin 1991)), can be compared with real-world data (Turner 1989; Gustafson and Parker 1992; Keitt, Urban, and Milne 1997, e.g.). Through modeling and empirical validation, such approaches are employed to explore general landscape structures (With and Crist 1995; With, Gardner, and Turner 1997; Saura and Martínez-Millán 2000; Riitters et al. 2007) to better understand processes related to land-cover patterns, habitat loss, influences on animal movement, species distribution, and the maintenance of landscape functions (McIntyre and Wiens 1999).

In Table 1 we list empirical works that treat percolation-like transitions in the urban context, providing details on the data and the variables of interest, see also Fig. 1. In the urban context, most works use the (a) clustering distance, (b) some sort of density, or (c) attributes of network edges as control parameters. As order parameter, the (i) cluster size(s), (ii) respective entropy, or (iii) number of clusters are used. For instance, an examination of the population density in UK wards reveals a transition in population and built area when population density varies (Arcaute et al. 2015). The authors suggest to consider this transition a proxy for defining cities. By varying the clustering distance, (Behnisch et al. 2019) demonstrate that the building locations in Germany form a giant cluster at the critical distance, which is related to the typical distance between neighboring buildings (Rozenfeld et al. 2008, see also). Yet, (Nam and Eom 2025) utilize population density in spatial units within cities to investigate how the number of clusters changes as the occupancy probability varies. (Bian, Yeh, and Zhang 2024) demonstrate the importance of percolating green corridors in cities. Further papers include (Molinero, Murcio, and Arcaute 2017; Piovani, Molinero, and Wilson 2017; Shreevastava, Rao, and McGrath 2019; Cao et al. 2020; Du et al. 2020, e.g.). Overall, the works represent examples in which understanding critical urban behavior can be a decisive data-driven approach to characterizing the morphology and functioning of the urban fabric.

Percolation can also be employed for modeling of cities and their evolution (Bitner, Hołyst, and Fiałkowski 2009; Fragkias and Seto 2009; Murcio, Sosa-Herrera, and Rodriguez-Romo 2013, e.g.). The percolation front represents a special form of percolation. If the occupancy probability changes along space, then also percolation happens along this gradient – with a so-called percolation front around the critical concentration. This idea is explored for cities, where a radial gradient is combined with correlated noise, leading to clusters that resemble cities (H. Makse, Havlin, and Stanley 1995; H. A. Makse et al. 1998).

Another domain where percolation-like transitions are explored is traffic dynamics. For example, the car speed threshold that separates isolated local flows from global flows in a road network serves to characterize traffic

Table 1. Selection of empirical studies investigating percolation-like transitions in the urban context. For each reference in the first column, we list the data analyzed as well as the order and control parameters. In all cases, the control parameter represents a threshold.

Authors	*Data*	*Order parameter*	*Control parameter*
Arcaute et al. (2015)	population density	largest cluster area, population	population density
Arcaute et al. (2016)	road network	clusters sizes	clustering distance
Fluschnik et al. (2016)	land-cover	average cluster size	clustering distance
Molinero, Murcio, and Arcaute (2017)	road network	entropy of cluster sizes	angle between roads
Piovani, Molinero, and Wilson (2017)	road network	entropy of cluster sizes	edge length
Huynh et al. (2018)	bus stops locations	largest cluster size, area	clustering distance
Behnisch et al. (2019)	building locations	cluster sizes	clustering distance
Shreevastava, Rao, and McGrath (2019)	land surface temperature	number of clusters	temperature
Cao et al. (2020)	nighttime light, population, road network	entropy of cluster areas	population density, road intersection density; day/night value
Du et al. (2020)	air quality index	fraction of the largest cluster	air quality index
Tapia-McClung, Montejano-Escamilla, and Caudillo-Cos (2023)	road network	entropy of cluster areas	clustering distance
Nam and Eom (2025)	population density	number of clusters	occupancy probability

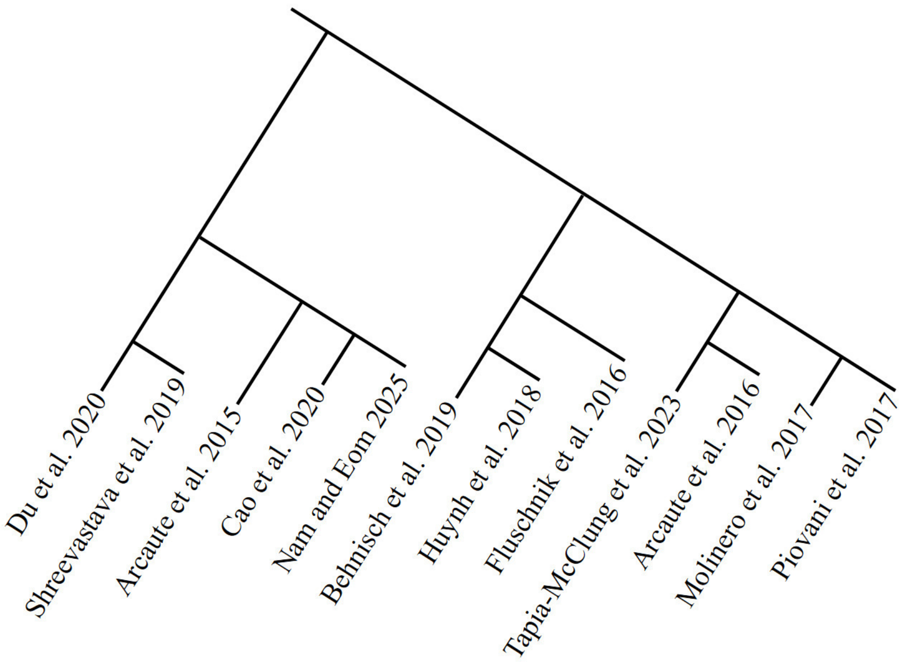

Figure 1. Hierarchical clustering (cladogram) of papers analyzing percolation-like transitions in the urban context. We processed the specific information (data, order parameter, control parameter), as detailed in Table 1, with TF-IDF vectorization and cosine similarity methods. E.g. Du et al. (2020) is more similar to Shreevastava, Rao, and McGrath (2019) than to Arcaute et al. (2015).

efficiency (D. Li et al. 2015). Studies show that the volume (F. Wang et al. 2015) and the duration of traffic flow (Zeng et al. 2019) affect the characteristics of this transition, and that the relationship between the size of the residential area and the width of the roads influences traffic congestion (Utami et al. 2020). (Ambühl, Menendez, and González 2023) find correlations between traffic congestion percolation and average flow through the network. (R. Wang, Wang, and Li 2023) report that all mobility networks undergo abrupt transitions when reaching a universal critical threshold. As demonstrated by (Verbavatz and Barthelemy 2021), one-way roads can lead to a percolation-like transition belonging to a new universality class. (Cogoni and Busonera 2021) find stable traffic patterns that persist at specific times of the day during the week and on weekends. (Ebrahimabadi et al. 2023) describe that the distribution of the supply and demand sources within the urban landscape can exert a significant influence on the critical traffic rate within the city. In addition, (C. Fan, Jiang, and Mostafavi 2020) combined an epidemic model with percolation ideas to predict flooding in urban networks. Further research on traffic dynamics includes (Talebpour, Mahmassani, and Hamdar 2017; Guo et al. 2018; Olmos et al. 2018; Serok et al. 2019; Chen et al. 2025, e.g.).

Percolation ideas are also employed to analyze forests and vegetation. In particular, (Saravia, Doyle, and Bond-Lamberty 2018) and (Taubert et al. 2018) show that regions with the highest rates of deforestation are closer to a critical threshold of fragmentation, and if fragmentation continues, consequences such as species loss and degradation of ecosystem services cannot be reversed. Similarly, some vegetation patterns in saturated environments, such as rain-forest, may benefit from being close to criticality (Villegas et al. 2024). In relation to forest fires, (Perestrelo et al. 2022) propose a percolation model to predict the phase transition during fire regimes.

Studies of settlement or vegetation connectivity can be subsumed under *landscape criticality*. Research on percolation theory is still progressing and advances are being made in various directions (Araújo et al. 2014; Saberi 2015; J. Fan, Meng, and Saberi 2019; M. Li et al. 2021; Sun et al. 2023; Schorcht et al. 2025, e.g.). Percolation theory provides a unifying framework for analyzing phase transitions in urban and landscape systems. From traffic resilience and urban sprawl to vegetation dynamics, critical thresholds offer actionable insights. While percolation theory is primarily based on randomness, real-world systems usually exhibit more complex (spatial) properties. Future research could be dedicated to bridge these two and to a better understanding of their differences.

Acknowledgements

We thank A.-K. Brenner, J.W. Kantelhardt, H. Samaniego, and W. Xu for usefull comments. R.L. Fagundes and Y. Li acknowledge financial support from the German Research Foundation (DFG) for the UPon (#451083179) project. R.L. Fagundes and D. Rybski thank Leibniz Association for funding the project "Landscape Criticality in the Anthropocene – Biodiversity, Renewables and Settlements" (CriticaL). D. Rybski additionally appreciates financial support through the DFG project Gropius (#511568027). F. L. Ribeiro thanks CNPq (grant numbers 403139/2021-0 and 424686/2021-0) and Fapemig (grant number APQ-00829-21 and APQ-06541-24) for financial support.

Submitted: September 01, 2025 AEDT. Accepted: November 25, 2025 AEDT. Published: December 12, 2025 AEDT.

REFERENCES

Ambühl, L., M. Menendez, and M. C. González. 2023. "Understanding Congestion Propagation by Combining Percolation Theory with the Macroscopic Fundamental Diagram." *Commun. Phys.* 6 (26). https://doi.org/10.1038/s42005-023-01144-w.

Araújo, N., Peter Grassberger, Byungnam Kahng, K. J. Schrenk, and Robert M. Ziff. 2014. "Recent Advances and Open Challenges in Percolation." *Eur. Phys. J. Spec. Top.* 223:2307–21. https://doi.org/10.1140/epjst/e2014-02266-y.

Arcaute, Elsa, Erez Hatna, Peter Ferguson, Hyejin Youn, Anders Johansson, and Michael Batty. 2015. "Constructing Cities, Deconstructing Scaling Laws." *J. R. Soc. Interface* 12 (102): 20140745. https://doi.org/10.1098/rsif.2014.0745.

Arcaute, Elsa, Carlos Molinero, Erez Hatna, Roberto Murcio, Camilo Vargas-Ruiz, A. Paolo Masucci, and Michael Batty. 2016. "Cities and Regions in Britain through Hierarchical Percolation." *R. Soc. Open Sci.* 3 (4): 150691. https://doi.org/10.1098/rsos.150691.

Balberg, Isaac. 2009. "Continuum Percolation." In *Encyclopedia of Complexity and Systems Science*, 1443–75. New York, NY: Springer New York. https://doi.org/10.1007/978-0-387-30440-3_95.

Behnisch, M., M. Schorcht, S. Kriewald, and D. Rybski. 2019. "Settlement Percolation: A Study of Building Connectivity and Poles of Inaccessibility." *Landscape Urban Plan.* 191:103631. https://doi.org/10.1016/j.landurbplan.2019.103631.

Bian, F., A. G. O. Yeh, and J. Zhang. 2024. "Percolating Spatial Scale Effects on the Landscape Connectivity of Urban Greenspace Network in Beijing, China." *Landsc. Ecol. Eng.* 20:33–51. https://doi.org/10.1007/s11355-023-00578-2.

Bitner, Agnieszka, Robert Hołyst, and Marcin Fiałkowski. 2009. "From Complex Structures to Complex Processes: Percolation Theory Applied to the Formation of a City." *Phys. Rev. E* 80:037102. https://doi.org/10.1103/PhysRevE.80.037102.

Brockmann, Dirk. 2018. "Barista's Secret." https://www.complexity-explorables.org/explorables/baristas-secret/.

Bunde, Armin, and Shlomo Havlin. 1991. *Fractals and Disordered Systems*. Springer Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-84868-1.

Cao, Wenpu, Lei Dong, Lun Wu, and Yu Liu. 2020. "Quantifying Urban Areas with Multi-Source Data Based on Percolation Theory." *RSE* 241:111730. https://doi.org/10.1016/j.rse.2020.111730.

Chen, Rui, Jiazhen Liu, Yong Li, and Yuming Lin. 2025. "Unveiling Multistability in Urban Traffic through Percolation Theory and Network Analysis." *Entropy* 27 (7). https://doi.org/10.3390/e27070668.

Cogoni, Marco, and Giovanni Busonera. 2021. "Stability of Traffic Breakup Patterns in Urban Networks." *Phys. Rev. E* 104:L012301. https://doi.org/10.1103/PhysRevE.104.L012301.

Du, Ruijin, Jingjing Li, Gaogao Dong, Lixin Tian, Ting Qing, Guochang Fang, and Yujuan Dong. 2020. "Percolation Analysis of Urban Air Quality: A Case in China." *Phys. A* 541:123312. https://doi.org/10.1016/j.physa.2019.123312.

Ebrahimabadi, Sasan, Ali Hosseiny, Jingfang Fan, and Abbas Ali Saberi. 2023. "Geometry of Commutes in the Universality of Percolating Traffic Flows." *Phys. Rev. E* 108:054311. https://doi.org/10.1103/PhysRevE.108.054311.

Efros, A. L. 1986. *Physics and Geometry of Disorder: Percolation Theory*. Science for Everyone. Mir Publishers Moscow.

Fan, Chao, Xiangqi Jiang, and Ali Mostafavi. 2020. "A Network Percolation-Based Contagion Model of Flood Propagation and Recession in Urban Road Networks." *Sci. Rep.* 10 (1): 13481. https://doi.org/10.1038/s41598-020-70524-x.

Fan, Jingfang, Jun Meng, and Abbas Ali Saberi. 2019. "Percolation Framework of the Earth's Topography." *Phys. Rev. E* 99 (February):022304. https://doi.org/10.1103/PhysRevE.99.022304.

Fluschnik, T., S. Kriewald, A. G. C. Ros, B. Zhou, D. E. Reusser, J. P. Kropp, and D. Rybski. 2016. "The Size Distribution, Scaling Properties and Spatial Organization of Urban Clusters: A Global and Regional Percolation Perspective." *ISPRS Int. J. Geo-Inf.* 5 (7): 110. https://doi.org/10.3390/ijgi5070110.

Fragkias, Michail, and Karen C. Seto. 2009. "Evolving Rank-Size Distributions of Intra-Metropolitan Urban Clusters in South China." *Comput. Environ. Urban Syst.* 33 (3): 189–99. https://doi.org/10.1016/j.compenvurbsys.2008.08.005.

Gardner, Robert H., Bruce T. Mime, Monica G. Turner, and Robert V. O'Neil. 1987. "Neutral Models for the Analysis of Broad-Scale Landscape Pattern." *Landscape Ecol.* 1 (1): 19–28. https://doi.org/10.1007/BF02275262.

Guo, Shengmin, Ruoqian Wu, Qingfeng Tong, Guanwen Zeng, Jian Yang, Long Chen, Tongyu Zhu, Weifeng Lv, and Daqing Li. 2018. "Is City Traffic Damaged by Torrential Rain?" *Phys. A* 503:1073–80. https://doi.org/10.1016/j.physa.2018.08.044.

Gustafson, Eric J., and George R. Parker. 1992. "Relationships between Landcover Proportion and Indices of Landscape Spatial Pattern." *Landsc. Ecol.* 7 (2): 101–10. https://doi.org/10.1007/BF02418941.

Huynh, Hoai Nguyen, Evgeny Makarov, Erika Fille Legara, Christopher Monterola, and Lock Yue Chew. 2018. "Characterisation and Comparison of Spatial Patterns in Urban Systems: A Case Study of U.S. Cities." *J. Comput. Sci.* 24:34–43. https://doi.org/10.1016/j.jocs.2017.12.001.

Isichenko, M. B. 1992. "Percolation, Statistical Topography, and Transport in Random Media." *Rev. Mod. Phys.* 64:961–1043. https://doi.org/10.1103/RevModPhys.64.961.

Keitt, T. H., D. L. Urban, and B. T. Milne. 1997. "Detecting Critical Scales in Fragmented Landscapes." *Conserv. Ecol.* 1 (1). http://www.consecol.org/vol1/iss1/art4/.

Li, Daqing, Bowen Fu, Yunpeng Wang, Guangquan Lu, Yehiel Berezin, H. Eugene Stanley, and Shlomo Havlin. 2015. "Percolation Transition in Dynamical Traffic Network with Evolving Critical Bottlenecks." *Proc. Natl. Acad. Sci. U.S.A* 112 (3): 669–72. https://doi.org/10.1073/pnas.1419185112.

Li, Ming, Run-Ran Liu, Linyuan Lü, Mao-Bin Hu, Shuqi Xu, and Yi-Cheng Zhang. 2021. "Percolation on Complex Networks: Theory and Application." *Phys. Rep.* 907:1–68. https://doi.org/10.1016/j.physrep.2020.12.003.

Makse, H., S. Havlin, and H. Eugene Stanley. 1995. "Modeling Urban Growth Patterns." *Nature* 337 (19): 608–12. https://doi.org/10.1038/377608a0.

Makse, Hernán A., José S. Andrade Jr., Michael Batty, Shlomo Havlin, and H. Eugene Stanley. 1998. "Modeling Urban Growth Patterns with Correlated Percolation." *Phys. Rev. E* 58 (6): 7054. https://doi.org/10.1103/PhysRevE.58.7054.

McIntyre, Nancy E., and John A. Wiens. 1999. "Interactions between Habitat Abundance and Configuration: Experimental Validation of Some Predictions from Percolation Theory." *Oikos* 86 (1): 129–37. https://doi.org/10.2307/3546577.

Meester, R., and R. Roy. 2008. *Continuum Percolation*. New York: Cambridge University Press. https://doi.org/10.1017/CBO9780511895357.

Mertens, Stephan, and Cristopher Moore. 2012. “Continuum Percolation Thresholds in Two Dimensions.” *Phys. Rev. E* 86:061109. https://doi.org/10.1103/PhysRevE.86.061109.

Molinero, Carlos, Roberto Murcio, and Elsa Arcaute. 2017. “The Angular Nature of Road Networks.” *Sci. Rep.* 7 (4312): 4312. https://doi.org/10.1038/s41598-017-04477-z.

Murcio, Roberto, Antonio Sosa-Herrera, and Suemi Rodriguez-Romo. 2013. “Second-Order Metropolitan Urban Phase Transitions.” *Chaos Soliton Fract.* 48:22–31. https://doi.org/10.1016/j.chaos.2013.01.001.

Nam, Yunwoo, and Young-Ho Eom. 2025. “Percolation Analysis of Spatiotemporal Distribution of Population in Seoul and Helsinki.” *Phys. Rev. E* 111:014305. https://doi.org/10.1103/PhysRevE.111.014305.

Olmos, Luis E., Serdar Çolak, Sajjad Shafiei, Meead Saberi, and Marta C. González. 2018. “Macroscopic Dynamics and the Collapse of Urban Traffic.” *Proc. Natl. Acad. Sci. U.S.A* 115 (50): 12654–61. https://doi.org/10.1073/pnas.1800474115.

Perestrelo, Sara Aleixo, Maria Clara Grácio, Nuno de Almeida Ribeiro, and Luís Mário Lopes. 2022. “A Multi-Scale Network with Percolation Model to Describe the Spreading of Forest Fires.” *Mathematics* 10 (4): 588. https://doi.org/10.3390/math10040588.

Piovani, Duccio, Carlos Molinero, and Alan Wilson. 2017. “Urban Retail Location: Insights from Percolation Theory and Spatial Interaction Modeling.” *PLoS One* 12 (10): 1–13. https://doi.org/10.1371/journal.pone.0185787.

Riitters, Kurt H., Peter Vogt, Pierre Soille, Jacek Kozak, and Christine Estreguil. 2007. “Neutral Model Analysis of Landscape Patterns from Mathematical Morphology.” *Landsc. Ecol.* 22:1033–43. https://doi.org/10.1007/s10980-007-9089-3.

Rozenfeld, H. D., D. Rybski, J. S. Andrade Jr., M. Batty, H. E. Stanley, and H. A. Makse. 2008. “Laws of Population Growth.” *Proc. Natl. Acad. Sci. U. S. A.* 105 (48): 18702–7. https://doi.org/10.1073/pnas.0807435105.

Saberi, Abbas Ali. 2015. “Recent Advances in Percolation Theory and Its Applications.” *Phys. Rep.* 578:1–32. https://doi.org/10.1016/j.physrep.2015.03.003.

Saravia, Leonardo, Santiago Doyle, and Ben Bond-Lamberty. 2018. “Power Laws and Critical Fragmentation in Global Forests.” *Sci. Rep.* 8 (17766). https://doi.org/10.1038/s41598-018-36120-w.

Saura, S., and J. Martínez-Millán. 2000. “Landscape Patterns Simulation with a Modified Random Clusters Method.” *Landsc. Ecol.* 15 (7): 661–78. https://doi.org/10.1023/a:1008107902848.

Schorcht, Martin, Martin Behnisch, Anna-Katharina Brenner, Tobias Krüger, Renan L. Fagundes, Wenjing Xu, Larissa Beumer, Thomas Müller, and Diego Rybski. 2025. “Settlement Percolation: Global Maps of Critical Distances.” in prep.

Serok, Nimrod, Orr Levy, Shlomo Havlin, and Efrat Blumenfeld-Lieberthal. 2019. “Unveiling the Inter-Relations between the Urban Streets Network and Its Dynamic Traffic Flows: Planning Implication.” *Environ. Plan. B* 46 (7): 1362–76. https://doi.org/10.1177/2399808319837982.

Shreevastava, Anamika, P. Suresh C. Rao, and Gavan S. McGrath. 2019. “Emergent Self-Similarity and Scaling Properties of Fractal Intra-Urban Heat Islets for Diverse Global Cities.” *Phys. Rev. E* 100:032142. https://doi.org/10.1103/PhysRevE.100.032142.

Stauffer, D., and A. Aharony. 1994. *Introduction to Percolation Theory: Second Edition (2nd Ed.)*. Taylor & Francis. https://doi.org/10.1201/9781315274386.

Sun, Yu, Teng Liu, Shang Wang, Jun Meng, Yongwen Zhang, Saini Yang, Xiaosong Chen, et al. 2023. “Regional Greening as a ‘Positive’ Tipping Phenomenon.” *arXiv Preprint arXiv:2312.03963*. https://doi.org/10.48550/arXiv.2312.03963.

Talebpour, Alireza, Hani S. Mahmassani, and Samer H. Hamdar. 2017. "Effect of Information Availability on Stability of Traffic Flow: Percolation Theory Approach." *Transp. Res. Proc.* 23:81–100. https://doi.org/10.1016/j.trpro.2017.05.006.

Tapia-McClung, Rodrigo, Jorge Alberto Montejano-Escamilla, and Camilo Alberto Caudillo-Cos. 2023. "Defining City Boundaries through Percolation Theory: The Case of the Mexican Urban System." *Econ. Soc. Territ.* 23 (73): 753–78. https://doi.org/10.22136/est20232087.

Taubert, Franziska, Rico Fischer, Jürgen Groeneveld, Sebastian Lehmann, Michael Müller, Edna Roedig, Thorsten Wiegand, and Andreas Huth. 2018. "Global Patterns of Tropical Forest Fragmentation." *Nature* 554:519–22. https://doi.org/10.1038/nature25508.

Turner, Monica Goigel. 1989. "Landscape Ecology: The Effect of Pattern on Process." *Annu. Rev. Ecol. Syst.* 20:171–97. https://www.jstor.org/stable/2097089.

Utami, Fisca Dian, Dui Yanto Rahman, Desyana Olenka Margaretta, Euis Sustini, and Mikrajuddin Abdullah. 2020. "Landscape Geometry-Based Percolation of Traffic in Several Populous Cities around the World." *J. King Saud Univ. Sci.* 32 (6): 2701–9. https://doi.org/10.1016/j.jksus.2020.06.004.

Verbavatz, Vincent, and Marc Barthelemy. 2021. "From One-Way Streets to Percolation on Random Mixed Graphs." *Phys. Rev. E* 103:042313. https://doi.org/10.1103/PhysRevE.103.042313.

Villegas, P., T. Gili, G. Caldarelli, and A. Gabrielli. 2024. "Evidence of Scale-Free Clusters of Vegetation in Tropical Rainforests." *Phys. Rev. E* 109 (4): L042402. https://doi.org/10.1103/PhysRevE.109.L042402.

Wang, Feilong, Daqing Li, Xiaoyun Xu, Ruoqian Wu, and Shlomo Havlin. 2015. "Percolation Properties in a Traffic Model." *Europhys. Lett.* 112 (3): 38001. https://doi.org/10.1209/0295-5075/112/38001.

Wang, Ruoxi, Qi Wang, and Nan Li. 2023. "Percolation Transitions in Urban Mobility Networks in America's 50 Largest Cities." *Sustain. Cities Soc.* 91:104435. https://doi.org/10.1016/j.scs.2023.104435.

With, Kimberly A., and Thomas O. Crist. 1995. "Critical Thresholds in Species' Responses to Landscape Structure." *Ecology* 76 (8): 2446–59. https://doi.org/10.2307/2265819.

With, Kimberly A., Robert H. Gardner, and Monica G. Turner. 1997. "Landscape Connectivity and Population Distributions in Heterogeneous Environments." *Oikos* 78 (1): 151–69. https://doi.org/10.2307/3545811.

Zeng, Guanwen, Daqing Li, Shengmin Guo, Liang Gao, Ziyou Gao, H. Eugene Stanley, and Shlomo Havlin. 2019. "Switch between Critical Percolation Modes in City Traffic Dynamics." *Proc. Natl. Acad. Sci. U.S.A* 116 (1): 23–28. https://doi.org/10.1073/pnas.1801545116.