A Supplemental Information: Examples

A.1 Ranking flip threshold (what makes rankings change?)

Motivation. The main text shows when rankings are invariant. This example isolates the opposite case, when two places can swap order as the impedance shifts attention from near to far opportunities.

Place A has $N_{A,\text{near}}$ opportunities at cost a and $N_{A,\text{far}}$ at cost $b > a$. Place B has $N_{B,\text{near}}$ at a and $N_{B,\text{far}}$ at b. With the exponential impedance $f(t) = e^{-\theta t}$,

$$A_A(\theta) - A_B(\theta) = (N_{A,\text{near}} - N_{B,\text{near}})e^{-\theta a} + (N_{A,\text{far}} - N_{B,\text{far}})e^{-\theta b}. \tag{12}$$

If the near advantage favors A and the far advantage favors B,

$$N_{A,\text{near}} > N_{B,\text{near}}, \qquad N_{A,\text{far}} < N_{B,\text{far}}, \tag{13}$$

to ensure $\theta^* > 0$, require $(N_{B,\text{far}} - N_{A,\text{far}}) > (N_{A,\text{near}} - N_{B,\text{near}})$; then there is a unique threshold θ^* where the ranking flips,

$$\theta^* = \frac{1}{b-a} \ln\left(\frac{N_{B,\text{far}} - N_{A,\text{far}}}{N_{A,\text{near}} - N_{B,\text{near}}} \right). \tag{14}$$

For $\theta > \theta^*$ the near opportunities dominate and $A_A(\theta) > A_B(\theta)$, for $\theta < \theta^*$ the far opportunities dominate and $A_B(\theta) > A_A(\theta)$.

For the cumulative impedance with budget T, the ranking is piecewise by threshold: if $T < a$ both are zero, if $a \le T < b$ the near counts decide, if $T \ge b$ the near plus far counts decide.

A.2 When different impedances give almost the same answer

Motivation. Practitioners often care about a specific time range, for example typical peak travel times. Many common impedances place most of their weight on that range. If two places look similar within that range, their access scores will be nearly proportional, no matter which such impedance is used.

Let the times where another opportunity becomes reachable be $\{t_k\}$ up to budget T, and write the discrete representation

$$A_i(T; f) = \sum_{k: t_k \le T} N_i(t_k)\, \Delta f_k, \qquad \Delta f_k = f(t_k) - f(t_{k+1}), \quad f(\infty) = 0. \tag{15}$$

Fix a time range of interest $[a, b] \subset [0, T]$ and split the sum into inside and outside that range,

$$A_i(T; f) = \underbrace{\sum_{t_k \in [a,b]} N_i(t_k)\, \Delta f_k}_{\text{in range}} + \underbrace{\sum_{t_k \in / [a,b]} N_i(t_k)\, \Delta f_k}_{\text{outside range}}. \tag{16}$$

When an impedance concentrates on a time range where the places look alike, different impedances make little difference to the comparison. More formally:

If most of the impedance weight lies in $[a, b]$,

$$\sum_{t_k \in\!\!/ [a,b]} \Delta f_k \leq \varepsilon \quad \text{with small } \varepsilon, \tag{17}$$

and the two places have approximately proportional cumulative curves on that same range,

$$N_A(t) \approx \alpha N_B(t) \quad \forall t \in [a, b], \tag{18}$$

then their scores are nearly proportional,

$$A_A(T; f) \approx \alpha\, A_B(T; f). \tag{19}$$

This leaves a discrepancy no larger than ε times a bound on the cumulative counts.