

TRANSPORT FINDINGS

Access Surplus: Valuing Accessibility by Integrating Opportunity Supply and Willingness to Pay

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Findings

We introduce Access Surplus as a welfare measure that frames accessibility in a market-like form: the inverse cumulative cost to reach the next opportunity is the ‘supply,’ and the willingness to pay for one more choice is the ‘demand.’ The area where demand exceeds supply, up to a natural stop point, is Access Surplus. The metric avoids arbitrary cut-offs, is additive over residents, links clearly to project effects, and stays transparent when only origin–destination times and counts are available.

1. Questions

Most accessibility indices either count opportunities within a generalised cost cut-off, or weight them by an impedance function. They are simple, yet cut-offs are arbitrary, and fixed weights are hard to defend (Hansen 1959; Weibull 1976; Geurs and Wee 2004; Wu and Levinson 2020).

Utility-based indicators grounded in discrete choice use the logsum as an accessibility measure and derive consumer surplus for appraisal (Geurs, Wee, and Rietveld 2013; Niemeier 1997; Handy and Niemeier 1997). Logsum measures are welfare-consistent, but they rely on a utility specification, a defined choice set, and a common scale across models (Williams 1977; Small and Rosen 1981; Train 2009). Practice also uses the ‘rule of half,’ which can be read as an accessibility interpretation of user benefits as an alternative to Logsum measures (Fosgerau and Pilegaard 2021). Recent work improves cumulative opportunity metrics by introducing diminishing returns to additional nearby opportunities, addressing over-counting when each extra destination is weighted equally (Roper, Ng, and Pettit 2023). What remains missing is a welfare index that operates on the ranked opportunity set, keeps option value and generalised cost as separate objects with common units, aggregates cleanly across people, and can align with, yet not depend on, a particular choice model. This paper fills that gap by defining a surplus of access while avoiding arbitrary distance cut-offs.

We ask whether a measure can value the marginal option in generalised cost units (time or money, including private or social costs (Cui and Levinson 2018)), avoid cut-offs, aggregate cleanly across residents, and remain

Table 1. Nomenclature

Symbol	Meaning	Units
$A_i(t)$	Primal (Hansen) access: opportunities reachable within t from i	opportunity
$\mathbb{A}_i$	Access Surplus at origin i	opportunity·minutes
$\mathbb{A}$	Aggregate Access Surplus, $\sum_i P_i \mathbb{A}_i$	opportunity·minutes
c_{ij}	Generalised cost from origin i to destination j	minutes (or cost)
$D_i(q)$	WTP in time for the q -th extra option (non-increasing in q)	minutes
$f_i^A(c_{ij})$	Induced weight $[D_i(\text{rank}_i(j)) - c_{ij}]_+$	opportunity·minutes
J_i	Number of opportunities considered from i	integer
O_j	Opportunity size at j	opportunity
P_i	Population at origin i	persons
q	Rank (index) of an opportunity	integer
q_i^*	Last rank with $D_i(q) \geq S_i(q)$	integer
$\text{rank}_i(j)$	Rank of j by increasing c_{ij} from i	integer
$S_i(q)$	Dual access: time to reach the q -th opportunity (supply)	minutes
VOT	Value of time, to monetise minutes	money per minute

transparent. We introduce access surplus ($\mathbb{A}$) as the net area between a decreasing willingness-to-pay-by-rank curve, $D_i(q)$, and the dual cumulative opportunity cost curve, $S_i(q)$.

2. Methods

Primal (Hansen) cumulative opportunities. Perhaps the most common access measure is cumulative opportunities (Hansen 1959):

$$A_i(t) = \sum_j O_j f_t(c_{ij}), \quad f_t(c) = \begin{cases} 1 & c \leq t, \\ 0 & c > t. \end{cases} \quad (1)$$

$A_i(t)$ is the number of opportunities reachable within t . Let c_{ij} be generalised cost from origin i to destination j , and let O_j be the size of the opportunity at j .

Dual access. Primal counts “how many by time.” But people often think in “how long until I can reach some number of places?” terms. Dual Access (Cui and Levinson 2020) captures that inverse idea by reading generalised cost (most typically travel time) off the sorted travel costs. Thus it is the cost of reaching a marginal additional opportunity. Concretely, the supply curve is just the q -th order statistic:

$$S_i(q) = c_{i(q)} \quad \text{for } q = 1, 2, \dots, J_i. \quad (2)$$

This profile is typically S-shaped in cities: quick gains nearby as area (and thus, typically, opportunities) increases with the square of distance or travel time, then slower gains as one nears the edge where the set of distinct places levels off and density drops. This is illustrated in Figure 1.

Demand $D_i(q)$ (value of one more option). As the choice set grows, the next distinct person or place adds less value.

Let $D_i(q)$ be the marginal willingness to pay in time (minutes) for the q -th extra option; $D_i(q)$ should be non-increasing in q . Unlike conventional demand models in transport, here we keep time disutility only in $S_i(q)$; $D_i(q)$ should capture the marginal positive value of option variety, not the negative disutility of travel time or monetary costs.

We can use a simple reduced-form, monotone sequence for the marginal value of the q -th extra option. For example, consider a generic power shape:

$$D_i(q) = D_i(1) q^{-\beta}, \quad D_i(1) > 0, \beta > 0. \quad (3)$$

Impose $D_i(q+1) \leq D_i(q)$ and $D_i(q) \geq 0$.

Though we talk of Supply and Demand, no direct market exchange is implied, noting that land markets implicitly reflect both opportunities and the willingness of travellers to reach them (Mann and Levinson 2024).

Access Surplus $\mathbb{A}$ (area between demand and supply). Supply and demand meet where the next option is just worth its cost. Define the stopping index:

$$q_i^* = \max\{q : D_i(q) \geq S_i(q)\} \quad (4)$$

Opportunities are discrete, so the welfare rectangle is naturally the sum over ranks, thus surplus is given by:

$$\mathbb{A}_i = \sum_{q=1}^{q_i^*} [D_i(q) - S_i(q)]_+, \quad [x]_+ = \max(x, 0). \quad (5)$$

A continuous form is provided in Appendix E.

Non-negativity. By construction, $\mathbb{A}$ is non-negative because only positive gaps between D and S are summed. Scarce or costly opportunity settings will yield small or zero A , but not negative values.

Units and interpretation. The unit of $\mathbb{A}$ is the product of the opportunity unit and the cost unit. For example, when opportunities are jobs and cost is time, the unit is “job·minutes”. In general we write “opportunity·minutes” as shorthand, recognising that the opportunity unit depends on the context. A money metric follows by multiplying the time component by a value of time (VOT).

To aggregate, sum $\mathbb{A}_i$ over origins, where P_i is the number of people at each origin:

$$\mathbb{A} = \sum_i^I P_i \mathbb{A}_i. \quad (6)$$

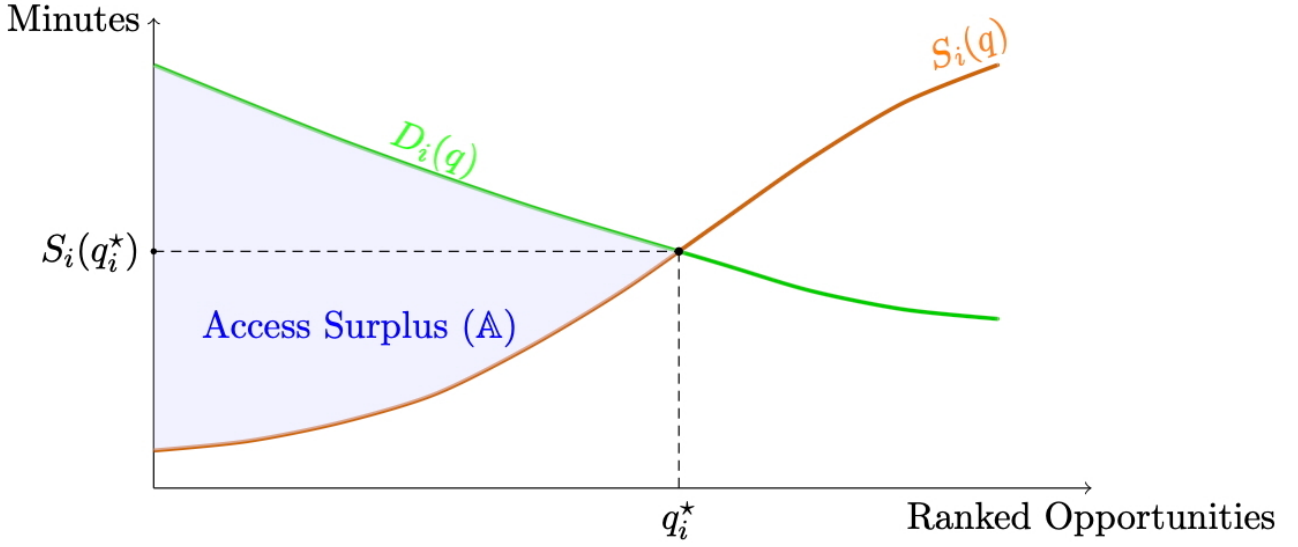


Figure 1. Access Surplus (shaded) is the area where willingness to pay by rank, $(D_i(q))$, exceeds the inverse cumulative cost (Dual Access), $(S_i(q))$, up to (q_i^*) , with costs in minutes.

Supplement roadmap. Extended details on this approach are in the Appendix: estimating $D(q)$ (Section A), recovering $D(q)$ from observed costs (Section B), separating option value from generalised cost (Section D), and the link between access surplus and welfare link and logsum (Section E).

Edges and shocks. We cap $S_i(q)$ at the last strictly increasing rank to avoid flat tails at the edge. A network improvement or local densification shifts $S_i(\cdot)$ downward (or flattens it), increasing $\mathbb{A}_i$ by widening the wedge between D_i and S_i . Preference shifts act through $D_i(\cdot)$.

3. Findings

Worked example. Think of a person looking for a lunch place. The supply $S_i(q)$ is the travel time to the q -th closest place, read from sorted door-to-door times. The demand $D_i(q)$ is the extra time the person would accept *to add one more opportunity (choice) to the set*. Early additions have high value, later ones add less.

The stopping rank is where demand first falls below supply, so

$$q_i^* = \max \{ q : D_i(q) \geq S_i(q) \} = 3. \quad (7)$$

The discrete Access Surplus sums the positive gaps up to q_i^* :

$$\begin{aligned} \mathbb{A}_i &= \sum_{q=1}^{q_i^*} [D_i(q) - S_i(q)]_+ = 10 + 4 + 0 \\ &= 14 \text{ opportunity} \cdot \text{minutes}. \end{aligned} \quad (8)$$

Table 2. Toy example: supply $S_i(q)$ versus demand $D_i(q)$

Rank q	$S_i(q)$ (min.)	$D_i(q)$ (min.)	Margin $D_i(q) - S_i(q)$ (min.)	Positive part	Cumulative $\mathbb{A}_i$ (opportunity·min.)
1	4	14	10	10	10
2	6	10	4	4	14
3	8	8	0	0	14
4	11	6	-5	0	14
5	15	4	-11	0	14
6	20	3	-17	0	14
7	28	2	-26	0	14
8	40	1	-39	0	14
After 1-minute improvement at $q = 2$					15

Interpretation. The first nearby option is very valuable, the second still helps, the third adds little, and beyond that the extra travel time to unlock more options exceeds what the person is willing to trade. $D_i(q)$ is the value of variety, in minutes, for adding the q -th option, while $S_i(q)$ is the time cost to unlock it. If the network improves so that $S_i(2)$ drops from 6 to 5 minutes, the surplus rises to 15; if a new place opens close by so that all $S_i(q)$ shift down by one rank, the surplus rises for the same $D_i(q)$.

Some attributes of the Access Surplus approach.

- **Welfare without cut-offs, clear stop rule.** $\mathbb{A}$ avoids arbitrary thresholds, and stops where $D_i = S_i$. The trade-off between more options and more generalised cost is clear on a single plot.
- **Additive, comparable, easy to explain.** $\mathbb{A}_i$ sums over people without harmonising model scales. Units are transparent (opportunity·minutes, then money via VOT). We can apply a figure to show whether gains come from faster networks (left shift of S), denser nearby options (flatter S), or changes in D : an upward shift when people are willing to trade more cost for each option, a flatter curve when they continue to value later options, or a steeper curve when value declines more quickly.
- **Edges, density, and positive externalities.** As $O_i(t)$ saturates near the edge, $S_i(q)$ steepens; $\mathbb{A}$ shows diminishing returns to pushing outward. Because opportunities are often also people, agglomeration can shift S_i down and, in some cases, raise D_i .

How to use access surplus in practice. *Project appraisal:* compute $\mathbb{A}_i$ for each origin, aggregate $\mathbb{A} = \sum_i P_i \mathbb{A}_i$, then report $\Delta\mathbb{A}$ and monetise by multiplying the time unit by a value of time (Mann and Levinson 2025). *Equity:* examine the distribution of $\mathbb{A}_i$ across groups, or set a minimum $\mathbb{A}_i$ target for underserved areas. *Comparability:* alongside $\mathbb{A}$, also report a conventional cumulative or gravity index for continuity and benchmarking. *Interpretation:* decompose gains by whether S shifts left (faster network), S flattens (more nearby options), or D shifts (preferences for variety). In applications, Access Surplus can serve as the welfare-consistent primary index, with cumulative or gravity measures reported in parallel for continuity.

Future extensions might consider competition. When destinations have finite capacity and users compete, we might replace raw counts with effective capacity-weighted counts before inversion (for example, using two-step floating catchment (Shen 1998)). In that case, we would compute $S_i(q)$ on the effective opportunities, then recompute $\mathbb{A}$.

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REFERENCES

- Cui, Mengying, and David Levinson. 2018. "Full Cost Accessibility." *Journal of Transport and Land Use* 11 (1): 661–79.
- . 2020. "Primal and Dual Access." *Geographical Analysis* 52 (3): 452–74.
- Fosgerau, Mogens, and Ninette Pilegaard. 2021. "The Rule-of-a-Half and Interpreting the Consumer Surplus as Accessibility." In *International Encyclopedia of Transportation*, edited by Roger Vickerman, 1:237–41. Elsevier. <https://doi.org/10.1016/B978-0-08-102671-7.10045-4>.
- Geurs, Karst T., and Bert van Wee. 2004. "Accessibility Evaluation of Land-Use and Transport Strategies: Review and Research Directions." *Journal of Transport Geography* 12 (2): 127–40. <https://doi.org/10.1016/j.jtrangeo.2003.10.005>.
- Geurs, Karst T., Bert van Wee, and Piet Rietveld. 2013. "Discussing the Logsum as an Accessibility Indicator." In *NECTAR Conference*.
- Handy, Susan L., and Debbie A. Niemeier. 1997. "Measuring Accessibility: An Exploration of Issues and Alternatives." *Environment and Planning A* 29 (7): 1175–94. <https://doi.org/10.1068/a291175>.
- Hansen, Walter G. 1959. "How Accessibility Shapes Land Use." *Journal of the American Institute of Planners* 25 (2): 73–76. <https://doi.org/10.1080/01944365908978307>.
- Mann, Isaac, and David M Levinson. 2024. "Access-Based Cost-Benefit Analysis." *Journal of Transport Geography* 119:103952.
- . 2025. "Valuing Travel Time and Reliability from Emerging Connected Vehicle Data." *Economics of Transportation* 44:100427.
- Niemeier, Debbie A. 1997. "Accessibility: An Evaluation Using Consumer Welfare." *Transportation* 24 (4): 377–96. <https://doi.org/10.1023/A:1004914803019>.
- Roper, Josephine, Matthew Ng, and Christopher Pettit. 2023. "Incorporating Diminishing Returns to Opportunities in Access: Development of an Open-Source Walkability Index Based on Multi-Activity Accessibility." *Journal of Transport and Land Use* 16:361–87. <https://doi.org/10.5198/jtlu.2023.2308>.
- Shen, Qing. 1998. "Location Characteristics of Inner-City Neighborhoods and Employment Accessibility of Low-Wage Workers." *Environment and Planning B: Planning and Design* 25 (3): 345–65.
- Small, Kenneth A., and Harvey S. Rosen. 1981. "Applied Welfare Economics with Discrete Choice Models." *Econometrica* 49 (1): 105–30. <https://doi.org/10.2307/1911129>.
- Train, Kenneth E. 2009. *Discrete Choice Methods with Simulation*. 2nd ed. Cambridge: Cambridge University Press. <https://doi.org/10.1017/CBO9780511805271>.
- Weibull, Jörgen W. 1976. "An Axiomatic Approach to the Measurement of Accessibility." *Regional Science and Urban Economics* 6 (4): 357–79. [https://doi.org/10.1016/0166-0462\(76\)90031-4](https://doi.org/10.1016/0166-0462(76)90031-4).
- Williams, H. C. W. L. 1977. "On the Formation of Travel Demand Models and Economic Evaluation Measures of User Benefit." *Environment and Planning A* 9 (3): 285–344. <https://doi.org/10.1068/a090285>.
- Wu, Hao, and David Levinson. 2020. "Unifying Access." *Transportation Research Part D: Transport and Environment* 83:102355.

SUPPLEMENTARY MATERIALS

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