Supplementary Information
A review of the structure of street networks

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Definitions

1. Degree

The street network is described by a network $G = (V, E)$ where V is a set of N nodes and E the set of links between these nodes. The nodes represent the intersections and the links segments of roads between these intersections. The degree k of a node is the number of streets converging to it. A node of degree $k = 1$ is a dead-end, nodes of degree $k = 2$ are generally removed and nodes of degree 3, 4. (or more) represent typical intersections. The average degree is then simply given by

$$\bar{k} = \frac{1}{N}\sum_{i=1}^{N} k_i$$

where k_i is the degree of node i. In general, the number of nodes of degree k is denoted by $N(k)$ and the proportion of dead-ends reads then

$$p_1 = \frac{N(1)}{N}$$

and of $k = 4$ intersections:

$$p_4 = \frac{N(4)}{N}$$

2. Detour index

The detour index (or stretch factor) for a pair of nodes i and j is defined as (Aldous & Shun, 2010; Barthelemy, 2022)

$$Q(i,j) = \frac{d_r(i,j)}{d_e(i,j)}$$

where d_e is the Euclidean distance between i and j, and d_r is the route distance computed on the network. We then have

$$Q_{max} = \max_{i,j} Q(i,j)$$

We can also average over pairs of nodes at a given distance d and construct the detour profile

$$Q(d) = \frac{1}{E(d)} \sum_{i,j \text{ s.t. } d_e(i,j)=d} Q(i,j)$$

where $E(d)$ is the number of pairs of nodes at distance d.

3. Total and average length

The total length of the network is defined as (Barthelemy, 2022)

$$L = \sum_{e \in E} l(e)$$

total length $l(e)$ is the length of edge e. The average edge length is then

$$l_1 = \frac{1}{N} L$$

4. Spatial planarity ratio

The spatial Planarity Ratio, φ (Boeing, 2020) represents the ratio of the number i_n of nonplanar intersections (i.e., non-dead-end nodes in the nonplanar, three-dimensional, spatially-embedded graph) to the number i_p of planar intersections (i.e., edge crossings in the planar, two-dimensional, spatially-embedded graph):

$$\varphi = \frac{i_p}{i_n}$$

The (positive) quantity $i_p - i_n$ is then equal to the number of nonplanar edge crossings such as overpasses and underpasses in the network.

5. Fraction of one-way streets

The fraction p of one-way streets is defined as (Verbavatz & Barthelemy, 2021)

$$p = \frac{L_1}{L}$$

where L_1 is the total length of one-way streets and L the total length of the network.

6. Betweenness centrality

An interesting quantity, first discussed in the context of non-spatial network (Freeman, 1977) is the betweenness centrality (BC). The betweenness centrality (BC) of a node i is defined as (Freeman, 1977)

$$g(i) = \frac{1}{(N-1)(N-2)} \sum_{s,t \neq i} \frac{\sigma_i(s,t)}{\sigma(s,t)}$$

where $\sigma(s,t)$ is the number of shortest paths from s to t, and $\sigma_i(s,t)$ is the number of such shortest path that go through the node i (and a similar definition for the BC of edges). The normalization (here chosen as the number of pairs of nodes different from i) can be slightly different according to different authors.

Bibliography

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